

6.003: Signals and Systems

Continuous-Time Systems

September 20, 2011

Multiple Representations of Discrete-Time Systems

Discrete-Time (DT) systems can be represented in different ways to more easily address different types of issues.

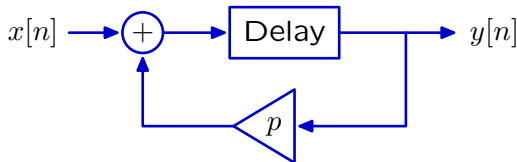
Verbal descriptions: preserve the rationale.

“Next year, your account will contain p times your balance from this year plus the money that you added this year.”

Difference equations: mathematically compact.

$$y[n+1] = x[n] + py[n]$$

Block diagrams: illustrate signal flow paths.



Operator representations: analyze systems as polynomials.

$$(1 - p\mathcal{R})Y = \mathcal{R}X$$

Multiple Representations of Continuous-Time Systems

Similar representations for Continuous-Time (CT) systems.

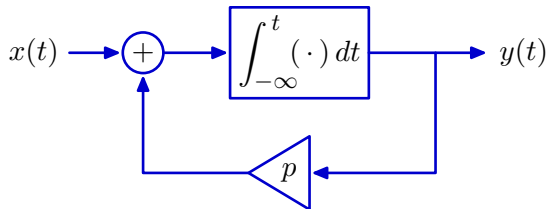
Verbal descriptions: preserve the rationale.

“Your account will grow in proportion to your balance plus the rate at which you deposit.”

Differential equations: mathematically compact.

$$\frac{dy(t)}{dt} = x(t) + py(t)$$

Block diagrams: illustrate signal flow paths.

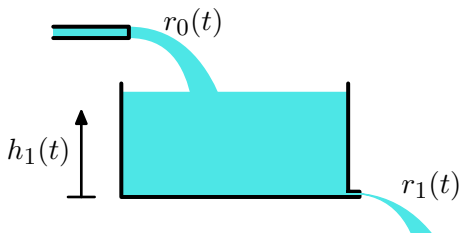


Operator representations: analyze systems as polynomials.

$$(1 - p\mathcal{A})Y = \mathcal{A}X$$

Differential Equations

Differential equations are mathematically precise and compact.



We can represent the tank system with a differential equation.

$$\frac{dr_1(t)}{dt} = \frac{r_0(t) - r_1(t)}{\tau}$$

You already know lots of methods to solve differential equations:

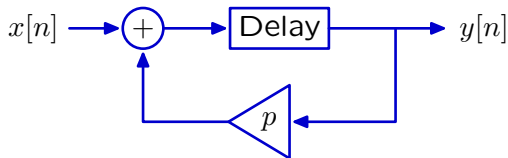
- general methods (separation of variables; integrating factors)
- homogeneous and particular solutions
- inspection

Today: new methods based on **block diagrams** and **operators**, which provide new ways to think about systems' behaviors.

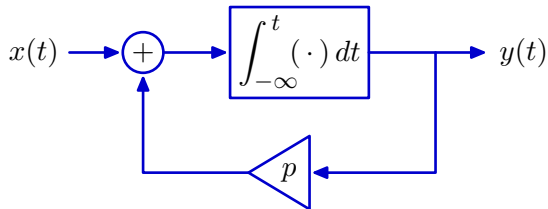
Block Diagrams

Block diagrams illustrate signal flow paths.

DT: adders, scalers, and delays – represent systems described by linear difference equations with constant coefficients.



CT: adders, scalers, and integrators – represent systems described by a linear differential equations with constant coefficients.



Delays in DT are replaced by integrators in CT.

Operator Representation

CT Block diagrams are concisely represented with the **$\mathcal{A}$ operator**.

Applying $\mathcal{A}$ to a CT signal generates a new signal that is equal to the integral of the first signal at all points in time.

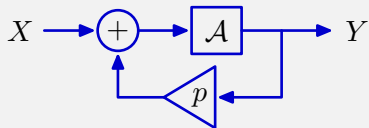
$$Y = \mathcal{A}X$$

is equivalent to

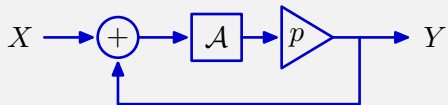
$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

for **all** time t .

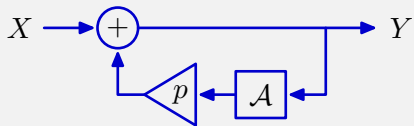
Check Yourself



$$\dot{y}(t) = \dot{x}(t) + py(t)$$

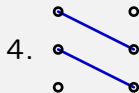
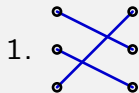


$$\dot{y}(t) = x(t) + py(t)$$



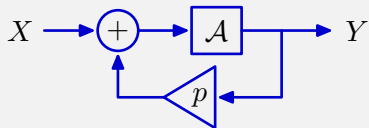
$$\dot{y}(t) = px(t) + py(t)$$

Which block diagrams correspond to which equations?

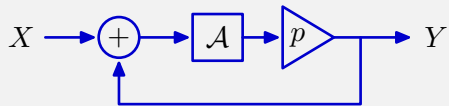


5. none

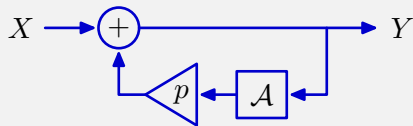
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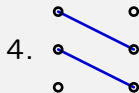
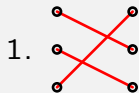
$$\dot{y}(t) = x(t) + py(t)$$



$$\dot{y}(t) = px(t) + py(t)$$

Which block diagrams correspond to which equations?

1

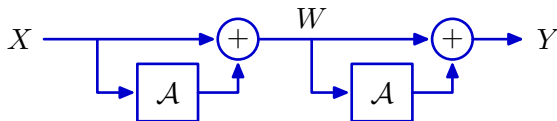


5. none

Evaluating Operator Expressions

As with $\mathcal{R}$, $\mathcal{A}$ expressions can be manipulated as polynomials.

Example:



$$w(t) = x(t) + \int_{-\infty}^t x(\tau) d\tau$$

$$y(t) = w(t) + \int_{-\infty}^t w(\tau) d\tau$$

$$y(t) = x(t) + \int_{-\infty}^t x(\tau) d\tau + \int_{-\infty}^t x(\tau) d\tau + \int_{-\infty}^t \left(\int_{-\infty}^{\tau_2} x(\tau_1) d\tau_1 \right) d\tau_2$$

$$W = (1 + \mathcal{A}) X$$

$$Y = (1 + \mathcal{A}) W = (1 + \mathcal{A})(1 + \mathcal{A}) X = (1 + 2\mathcal{A} + \mathcal{A}^2) X$$

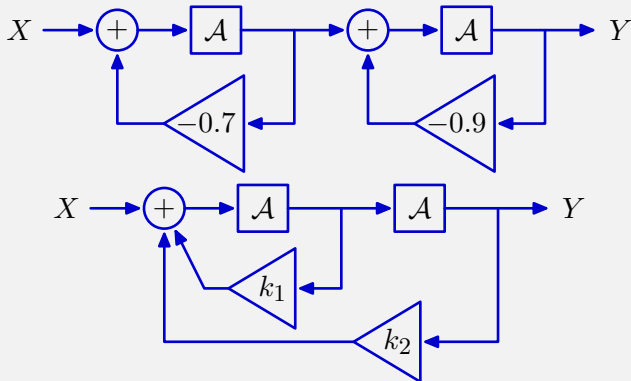
Evaluating Operator Expressions

Expressions in $\mathcal{A}$ can be manipulated using rules for polynomials.

- Commutativity: $\mathcal{A}(1 - \mathcal{A})X = (1 - \mathcal{A})\mathcal{A}X$
- Distributivity: $\mathcal{A}(1 - \mathcal{A})X = (\mathcal{A} - \mathcal{A}^2)X$
- Associativity: $\left((1 - \mathcal{A})\mathcal{A}\right)(2 - \mathcal{A})X = (1 - \mathcal{A})\left(\mathcal{A}(2 - \mathcal{A})\right)X$

Check Yourself

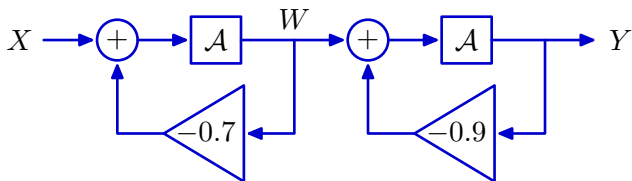
Determine k_1 so that these systems are “equivalent.”



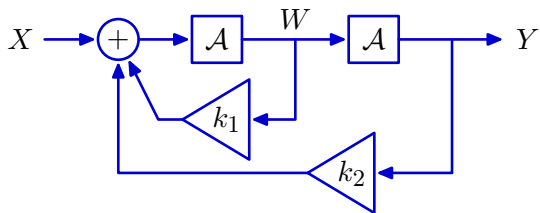
1. 0.7 2. 0.9 3. 1.6 4. 0.63 5. none of these

Check Yourself

Write operator expressions for each system.



$$\begin{aligned} W &= \mathcal{A}(X - 0.7W) & (1 + 0.7\mathcal{A})W &= \mathcal{A}X & (1 + 0.7\mathcal{A})(1 + 0.9\mathcal{A})Y &= \mathcal{A}^2 X \\ Y &= \mathcal{A}(W - 0.9Y) & (1 + 0.9\mathcal{A})Y &= \mathcal{A}W & (1 + 1.6\mathcal{A} + 0.63\mathcal{A}^2)Y &= \mathcal{A}^2 X \end{aligned}$$

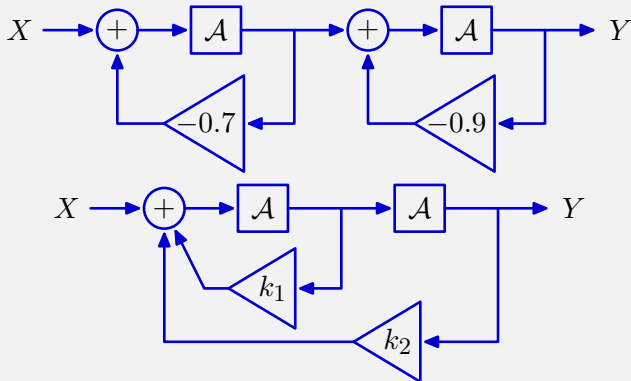


$$\begin{aligned} W &= \mathcal{A}(X + k_1 W + k_2 Y) & Y &= \mathcal{A}^2 X + k_1 \mathcal{A} Y + k_2 \mathcal{A}^2 Y \\ Y &= \mathcal{A} W & (1 - k_1 \mathcal{A} - k_2 \mathcal{A}^2)Y &= \mathcal{A}^2 X \end{aligned}$$

$$k_1 = -1.6$$

Check Yourself

Determine k_1 so that these systems are “equivalent.”

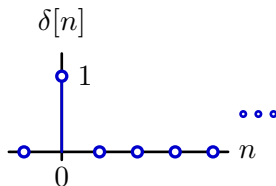


1. 0.7 2. 0.9 3. 1.6 4. 0.63 5. none of these

Elementary Building-Block Signals

Elementary DT signal: $\delta[n]$.

$$\delta[n] = \begin{cases} 1, & \text{if } n = 0; \\ 0, & \text{otherwise} \end{cases}$$



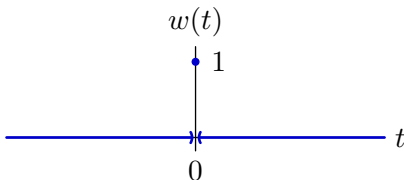
- simplest non-trivial signal (only one non-zero value)
- shortest possible duration (most “transient”)
- useful for constructing more complex signals

What CT signal serves the same purpose?

Elementary CT Building-Block Signal

Consider the analogous CT signal: $w(t)$ is non-zero only at $t = 0$.

$$w(t) = \begin{cases} 0 & t < 0 \\ 1 & t = 0 \\ 0 & t > 0 \end{cases}$$



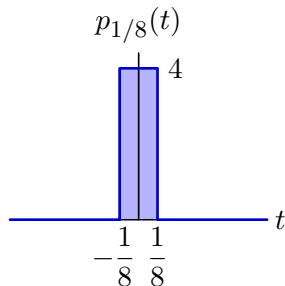
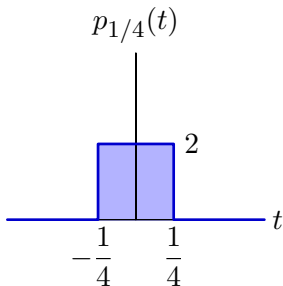
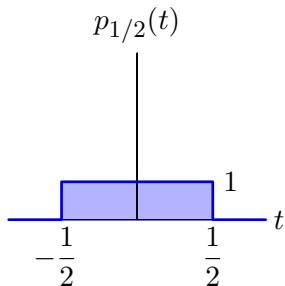
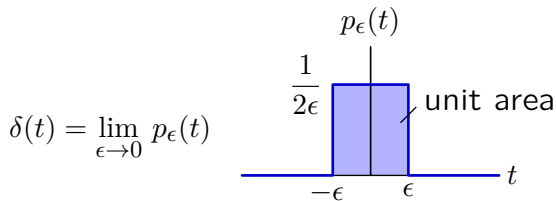
Is this a good choice as a building-block signal? **No**

$$w(t) \rightarrow \boxed{\int_{-\infty}^t (\cdot) dt} \rightarrow 0$$

The integral of $w(t)$ is zero!

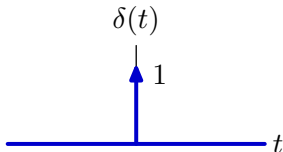
Unit-Impulse Signal

The unit-impulse signal acts as a pulse with unit area but zero width.



Unit-Impulse Signal

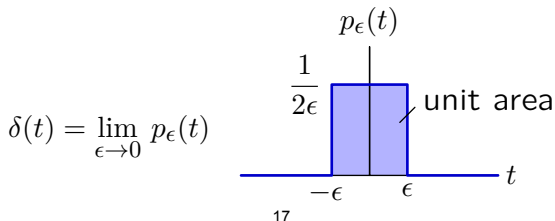
The unit-impulse function is represented by an arrow with the number **1**, which represents its area or “weight.”



It has two seemingly contradictory properties:

- it is nonzero only at $t = 0$, and
- its definite integral $(-\infty, \infty)$ is one!

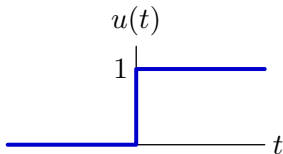
Both of these properties follow from thinking about $\delta(t)$ as a limit:



Unit-Impulse and Unit-Step Signals

The indefinite integral of the unit-impulse is the unit-step.

$$u(t) = \int_{-\infty}^t \delta(\lambda) d\lambda = \begin{cases} 1; & t \geq 0 \\ 0; & \text{otherwise} \end{cases}$$

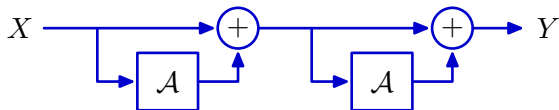


Equivalently



Impulse Response of Acyclic CT System

If the block diagram of a CT system has no feedback (i.e., no cycles), then the corresponding operator expression is “imperative.”



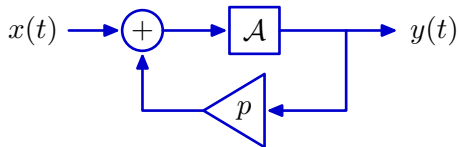
$$Y = (1 + \mathcal{A})(1 + \mathcal{A}) X = (1 + 2\mathcal{A} + \mathcal{A}^2) X$$

If $x(t) = \delta(t)$ then

$$y(t) = (1 + 2\mathcal{A} + \mathcal{A}^2) \delta(t) = \delta(t) + 2u(t) + tu(t)$$

CT Feedback

Find the impulse response of this CT system with feedback.



Method 1: find differential equation and solve it.

$$\dot{y}(t) = x(t) + py(t)$$

Linear, first-order difference equation with constant coefficients.

Try $y(t) = Ce^{\alpha t}u(t)$.

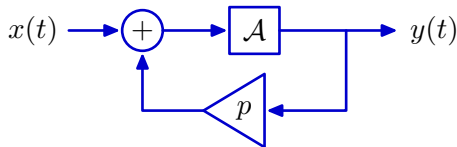
Then $\dot{y}(t) = \alpha Ce^{\alpha t}u(t) + Ce^{\alpha t}\delta(t) = \alpha Ce^{\alpha t}u(t) + C\delta(t)$.

Substituting, we find that $\alpha Ce^{\alpha t}u(t) + C\delta(t) = \delta(t) + pCe^{\alpha t}u(t)$.

Therefore $\alpha = p$ and $C = 1 \rightarrow y(t) = e^{pt}u(t)$.

CT Feedback

Find the impulse response of this CT system with feedback.



Method 2: use operators.

$$Y = \mathcal{A}(X + pY)$$

$$\frac{Y}{X} = \frac{\mathcal{A}}{1 - p\mathcal{A}}$$

Now expand in ascending series in $\mathcal{A}$:

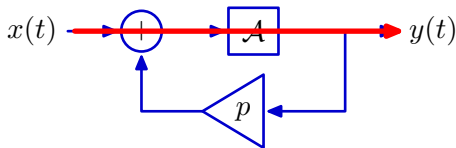
$$\frac{Y}{X} = \mathcal{A}(1 + p\mathcal{A} + p^2\mathcal{A}^2 + p^3\mathcal{A}^3 + \dots)$$

If $x(t) = \delta(t)$ then

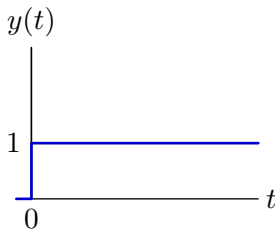
$$\begin{aligned} y(t) &= \mathcal{A}(1 + p\mathcal{A} + p^2\mathcal{A}^2 + p^3\mathcal{A}^3 + \dots) \delta(t) \\ &= (1 + pt + \frac{1}{2}p^2t^2 + \frac{1}{6}p^3t^3 + \dots) u(t) = e^{pt}u(t). \end{aligned}$$

CT Feedback

We can visualize the feedback by tracing each cycle through the cyclic signal path.

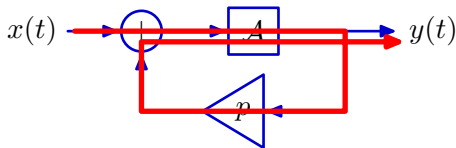


$$\begin{aligned}y(t) &= (\mathcal{A} + p\mathcal{A}^2 + p^2\mathcal{A}^3 + p^3\mathcal{A}^4 + \dots) \delta(t) \\&= (1 + pt + \frac{1}{2}p^2t^2 + \frac{1}{6}p^3t^3 + \dots) u(t)\end{aligned}$$

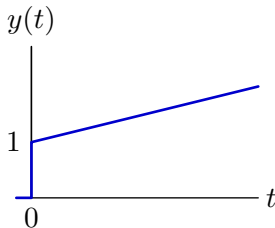


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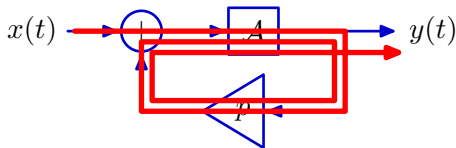


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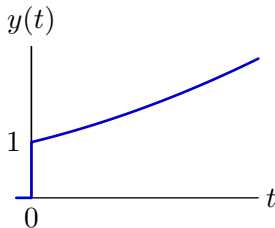


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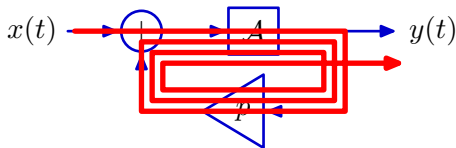


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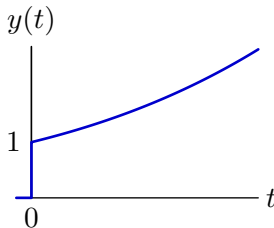


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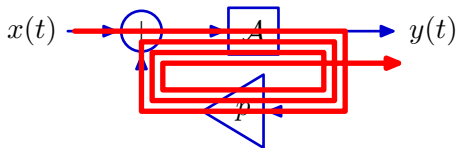


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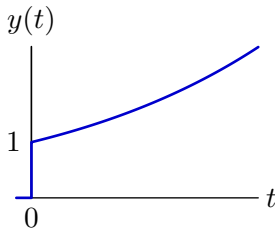


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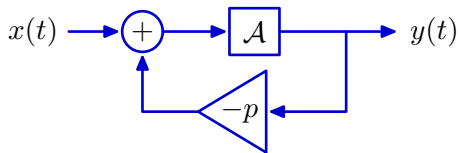


$$\begin{aligned} y(t) &= (\mathcal{A} + p\mathcal{A}^2 + p^2\mathcal{A}^3 + \textcolor{red}{p}^3\mathcal{A}^4 + \cdots) \delta(t) \\ &= (1 + pt + \frac{1}{2}p^2t^2 + \frac{1}{6}p^3t^3 + \cdots) u(t) = \textcolor{red}{e}^{pt}u(t) \end{aligned}$$



CT Feedback

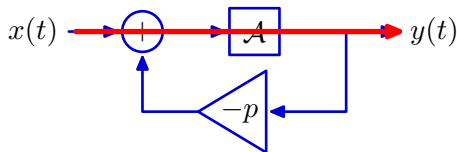
Making p negative makes the output converge (instead of diverge).



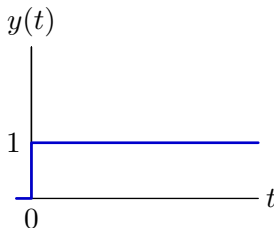
$$\begin{aligned} y(t) &= (\mathcal{A} - p\mathcal{A}^2 + p^2\mathcal{A}^3 - p^3\mathcal{A}^4 + \dots) \delta(t) \\ &= (1 - pt + \frac{1}{2}p^2t^2 - \frac{1}{6}p^3t^3 + \dots) u(t) \end{aligned}$$

CT Feedback

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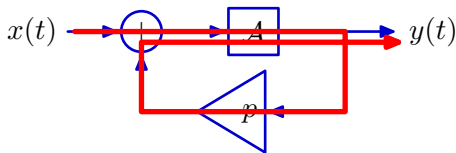


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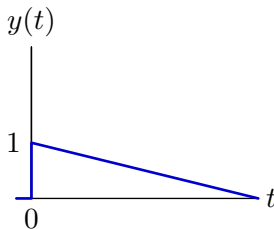


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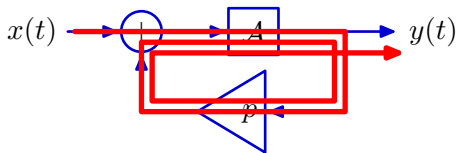


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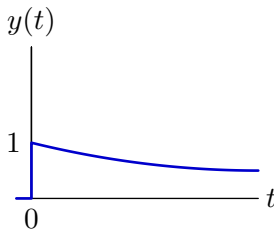


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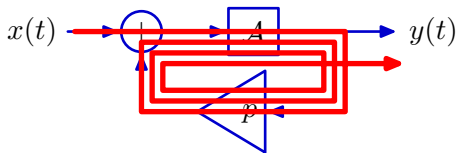


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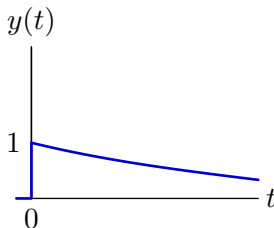


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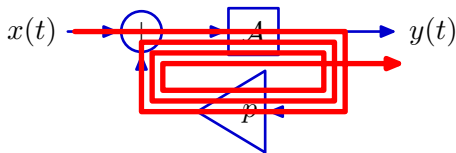


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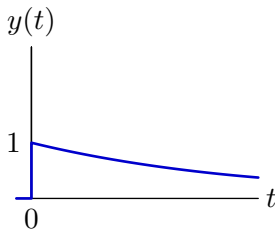


CT Feedback

Making p negative makes the output converge (instead of diverge).

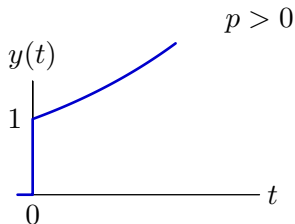
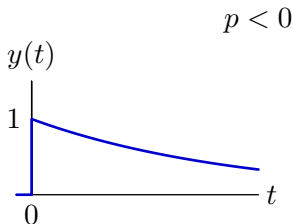
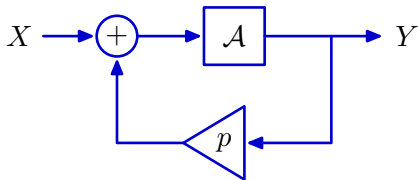


$$\begin{aligned} y(t) &= (\mathcal{A} - p\mathcal{A}^2 + p^2\mathcal{A}^3 - \textcolor{red}{p}^3\mathcal{A}^4 + \cdots) \delta(t) \\ &= (1 - pt + \frac{1}{2}p^2t^2 - \frac{1}{6}p^3t^3 + \cdots) u(t) = \textcolor{red}{e}^{-pt}u(t) \end{aligned}$$



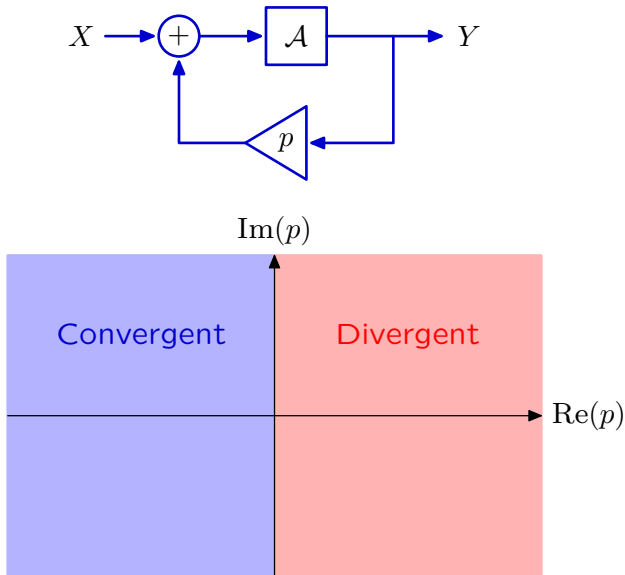
Convergent and Divergent Poles

The fundamental mode associated with p converges if $p < 0$ and diverges if $p > 0$.



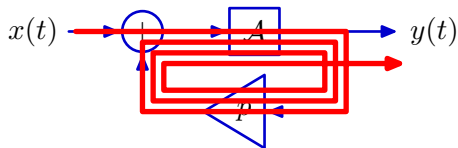
Convergent and Divergent Poles

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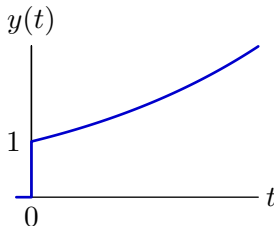


CT Feedback

In CT, each cycle adds a new integration.

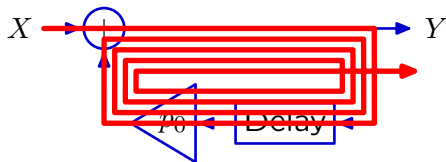


$$\begin{aligned} y(t) &= (\mathcal{A} + p\mathcal{A}^2 + p^2\mathcal{A}^3 + p^3\mathcal{A}^4 + \cdots) \delta(t) \\ &= (1 + pt + \frac{1}{2}p^2t^2 + \frac{1}{6}p^3t^3 + \cdots) u(t) = e^{pt}u(t) \end{aligned}$$

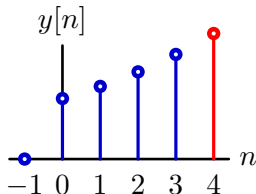


DT Feedback

In DT, each cycle creates another sample in the output.



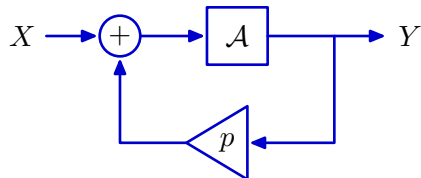
$$\begin{aligned} y[n] &= (1 + p\mathcal{R} + p^2\mathcal{R}^2 + p^3\mathcal{R}^3 + p^4\mathcal{R}^4 + \cdots) \delta[n] \\ &= \delta[n] + p\delta[n-1] + p^2\delta[n-2] + p^3\delta[n-3] + p^4\delta[n-4] + \cdots \end{aligned}$$



Summary: CT and DT representations

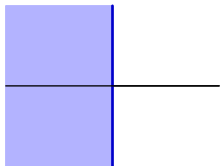
Many similarities and important differences.

$$\dot{y}(t) = x(t) + py(t)$$

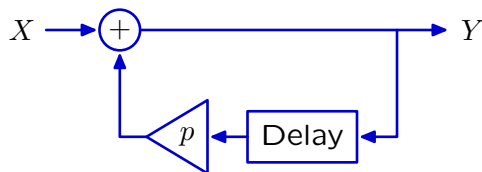


$$\frac{\mathcal{A}}{1 - p\mathcal{A}}$$

$$e^{pt}u(t)$$

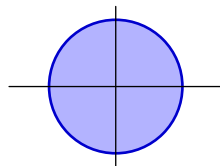


$$y[n] = x[n] + py[n - 1]$$



$$\frac{1}{1 - p\mathcal{R}}$$

$$p^n u[n]$$



Check Yourself

Which functionals represent convergent systems?

$$\frac{1}{1 - \frac{1}{4}\mathcal{R}^2}$$

$$\frac{1}{1 - \frac{1}{4}\mathcal{A}^2}$$

$$\frac{1}{1 + 2\mathcal{R} + \frac{3}{4}\mathcal{R}^2}$$

$$\frac{1}{1 + 2\mathcal{A} + \frac{3}{4}\mathcal{A}^2}$$

1. $\begin{array}{c} \checkmark \times \\ \checkmark \times \end{array}$

2. $\begin{array}{c} \checkmark \checkmark \\ \times \times \end{array}$

3. $\begin{array}{c} \checkmark \checkmark \\ \checkmark \checkmark \end{array}$

4. $\begin{array}{c} \checkmark \times \\ \times \checkmark \end{array}$

5. none of these

Check Yourself

$$\frac{1}{1 - \frac{1}{4}\mathcal{R}^2} = \frac{1}{(1 - \frac{1}{2}\mathcal{R})(1 + \frac{1}{2}\mathcal{R})}$$

both inside unit circle

✓

$$\frac{1}{1 - \frac{1}{4}\mathcal{A}^2} = \frac{1}{(1 - \frac{1}{2}\mathcal{A})(1 + \frac{1}{2}\mathcal{A})}$$

left & right half-planes

X

$$\frac{1}{1 + 2\mathcal{R} + \frac{3}{4}\mathcal{R}^2} = \frac{1}{(1 + \frac{1}{2}\mathcal{R})(1 + \frac{3}{2}\mathcal{R})}$$

inside & outside unit circle

X

$$\frac{1}{1 + 2\mathcal{A} + \frac{3}{4}\mathcal{A}^2} = \frac{1}{(1 + \frac{1}{2}\mathcal{A})(1 + \frac{3}{2}\mathcal{A})}$$

both left half plane

✓

Check Yourself

Which functionals represent convergent systems? 4

$$\frac{1}{1 - \frac{1}{4}\mathcal{R}^2}$$

$$\frac{1}{1 - \frac{1}{4}\mathcal{A}^2}$$

$$\frac{1}{1 + 2\mathcal{R} + \frac{3}{4}\mathcal{R}^2}$$

$$\frac{1}{1 + 2\mathcal{A} + \frac{3}{4}\mathcal{A}^2}$$

1. $\begin{array}{cc} \checkmark & \times \\ \checkmark & \times \end{array}$

2. $\begin{array}{cc} \checkmark & \checkmark \\ \times & \times \end{array}$

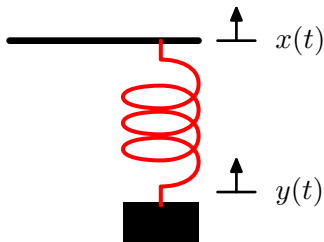
3. $\begin{array}{cc} \checkmark & \checkmark \\ \checkmark & \checkmark \end{array}$

4. $\begin{array}{cc} \checkmark & \times \\ \times & \checkmark \end{array}$

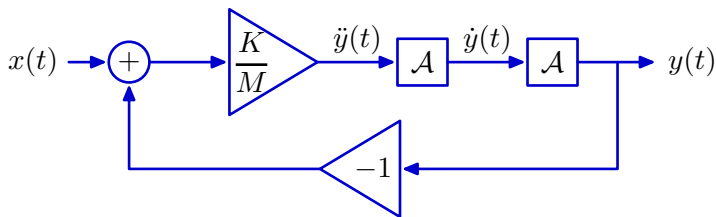
5. none of these

Mass and Spring System

Use the $\mathcal{A}$ operator to solve the mass and spring system.



$$F = K(x(t) - y(t)) = M\ddot{y}(t)$$



$$\frac{Y}{X} = \frac{\frac{K}{M}\mathcal{A}^2}{1 + \frac{K}{M}\mathcal{A}^2}$$

Mass and Spring System

Factor system functional to find the poles.

$$\frac{Y}{X} = \frac{\frac{K}{M}\mathcal{A}^2}{1 + \frac{K}{M}\mathcal{A}^2} = \frac{\frac{K}{M}\mathcal{A}^2}{(1 - p_0\mathcal{A})(1 - p_1\mathcal{A})}$$

$$1 + \frac{K}{M}\mathcal{A}^2 = 1 - (p_0 + p_1)\mathcal{A} + p_0p_1\mathcal{A}^2$$

The sum of the poles must be zero.

The product of the poles must be K/M .

$$p_0 = j\sqrt{\frac{K}{M}} \quad p_1 = -j\sqrt{\frac{K}{M}}$$

Mass and Spring System

Alternatively, find the poles by substituting $\mathcal{A} \rightarrow \frac{1}{s}$.

The poles are then the roots of the denominator.

$$\frac{Y}{X} = \frac{\frac{K}{M}\mathcal{A}^2}{1 + \frac{K}{M}\mathcal{A}^2}$$

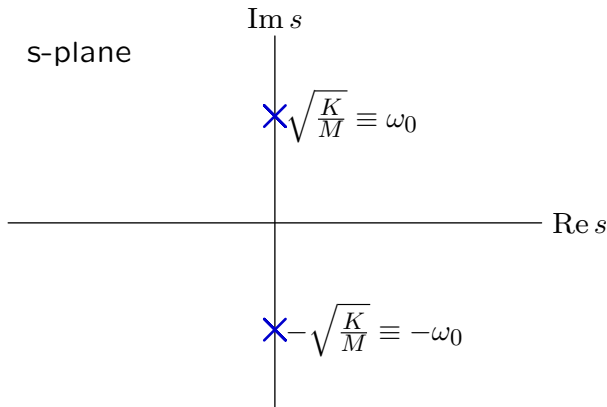
Substitute $\mathcal{A} \rightarrow \frac{1}{s}$:

$$\frac{Y}{X} = \frac{\frac{K}{M}}{s^2 + \frac{K}{M}}$$

$$s = \pm j\sqrt{\frac{K}{M}}$$

Mass and Spring System

The poles are complex conjugates.



The corresponding fundamental modes have complex values.

fundamental mode 1: $e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$

fundamental mode 2: $e^{-j\omega_0 t} = \cos \omega_0 t - j \sin \omega_0 t$

Mass and Spring System

Real-valued inputs always excite combinations of these modes so that the imaginary parts cancel.

Example: find the impulse response.

$$\begin{aligned}\frac{Y}{X} &= \frac{\frac{K}{M}\mathcal{A}^2}{1 + \frac{K}{M}\mathcal{A}^2} = \frac{\frac{K}{M}}{p_0 - p_1} \left(\frac{\mathcal{A}}{1 - p_0\mathcal{A}} - \frac{\mathcal{A}}{1 - p_1\mathcal{A}} \right) \\ &= \frac{\omega_0^2}{2j\omega_0} \left(\frac{\mathcal{A}}{1 - j\omega_0\mathcal{A}} - \frac{\mathcal{A}}{1 + j\omega_0\mathcal{A}} \right) \\ &= \frac{\omega_0}{2j} \underbrace{\left(\frac{\mathcal{A}}{1 - j\omega_0\mathcal{A}} \right)}_{\text{makes mode 1}} - \frac{\omega_0}{2j} \underbrace{\left(\frac{\mathcal{A}}{1 + j\omega_0\mathcal{A}} \right)}_{\text{makes mode 2}}\end{aligned}$$

The modes themselves are complex conjugates, and their coefficients are also complex conjugates. So the sum is a sum of something and its complex conjugate, which is real.

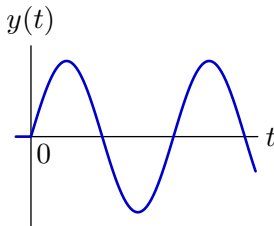
Mass and Spring System

The impulse response is therefore real.

$$\frac{Y}{X} = \frac{\omega_0}{2j} \left(\frac{\mathcal{A}}{1 - j\omega_0 \mathcal{A}} \right) - \frac{\omega_0}{2j} \left(\frac{\mathcal{A}}{1 + j\omega_0 \mathcal{A}} \right)$$

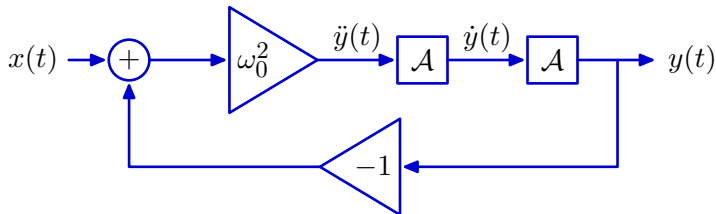
The impulse response is

$$h(t) = \frac{\omega_0}{2j} e^{j\omega_0 t} - \frac{\omega_0}{2j} e^{-j\omega_0 t} = \omega_0 \sin \omega_0 t; \quad t > 0$$



Mass and Spring System

Alternatively, find impulse response by expanding system functional.



$$\frac{Y}{X} = \frac{\omega_0^2 \mathcal{A}^2}{1 + \omega_0^2 \mathcal{A}^2} = \omega_0^2 \mathcal{A}^2 - \omega_0^4 \mathcal{A}^4 + \omega_0^6 \mathcal{A}^6 - + \dots$$

If $x(t) = \delta(t)$ then

$$y(t) = \omega_0^2 t - \omega_0^4 \frac{t^3}{3!} + \omega_0^6 \frac{t^5}{5!} - + \dots, \quad t \geq 0$$

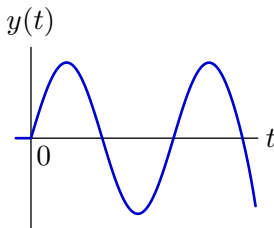
Mass and Spring System

Look at successive approximations to this infinite series.

$$\frac{Y}{X} = \frac{\omega_0^2 \mathcal{A}^2}{1 + \omega_0^2 \mathcal{A}^2} = \omega_0^2 \mathcal{A}^2 \sum_{l=0}^{\infty} \left(-\omega_0^2 \mathcal{A}^2 \right)^l$$

If $x(t) = \delta(t)$ then

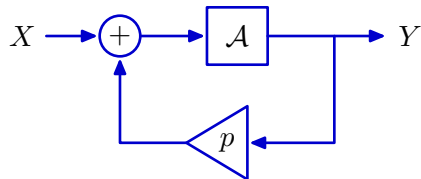
$$\begin{aligned} y(t) &= \sum_{l=0}^{\infty} \omega_0^2 \left(-\omega_0^2 \right)^l \mathcal{A}^{2l+2} \delta(t) \\ &= \omega_0^2 t - \omega_0^4 \frac{t^3}{3!} + \omega_0^6 \frac{t^5}{5!} - \omega_0^8 \frac{t^7}{7!} + \omega_0^{10} \frac{t^9}{9!} - + \cdots = \omega_0 \sin \omega_0 t \end{aligned}$$



Summary: CT and DT representations

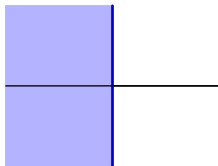
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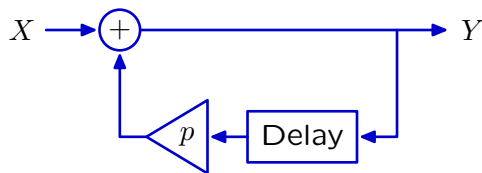


$$\frac{\mathcal{A}}{1 - p\mathcal{A}}$$

$$e^{pt}u(t)$$

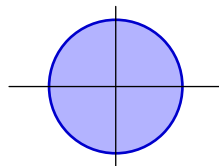


$$y[n] = x[n] + py[n - 1]$$



$$\frac{1}{1 - p\mathcal{R}}$$

$$p^n u[n]$$



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6.003 Signals and Systems

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