

Introduction and Optimization Problems

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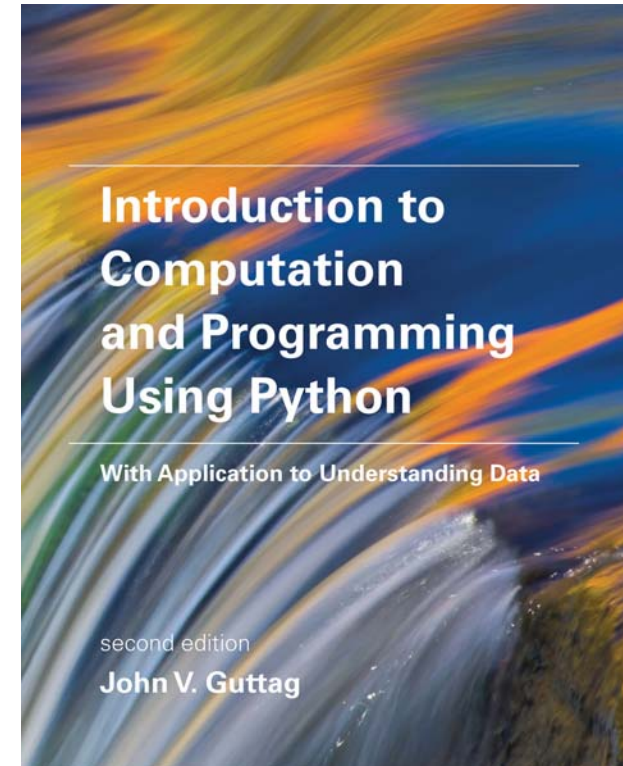
6.0002 Prerequisites

- Experience writing object-oriented programs in Python
 - Preferably in Python 3.5
- Familiarity with concepts of computational complexity
- Familiarity with some simple algorithms
- **6.0001** sufficient

Question 1

Some Administrative Things

- Problem sets
 - Programming problems designed to
 - Improve your programming skills
 - Help you learn the conceptual material
- Finger exercises
 - Very small programming problems designed to help you learn a single programming concept
- Reading assignments in textbook
 - Another take on and more details about material covered by lectures and problem sets
- Exam: based on above



How Does It Compare to 6.0001?

- Programming assignments a bit easier
 - Focus more on the problem to be solved than on programming
- Lecture content more abstract
- Lectures will be a bit faster paced
- Less about learning to program, more about dipping your toe into data science

Honing Your Programming Skills

- A few additional bits of Python
- Software engineering
- Using packages
- How do you get to Carnegie Hall?

Computational Models

- Using computation to help understand the world in which we live
- Experimental devices that help us to understand something that has happened or to predict the future



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- Optimization models
- Statistical models
- Simulation models

Relevant Reading for Today's Lecture

- Section 12.1
- Section 5.4 (lambda functions)

Computational Models

- Using computation to help understand the world in which we live
- Experimental devices that help us to understand something that has happened or to predict the future



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- *Optimization models*
- Statistical models
- Simulation models

What Is an Optimization Model?

- An objective function that is to be maximized or minimized, e.g.,
 - Minimize time spent traveling from New York to Boston
- A set of constraints (possibly empty) that must be honored, e.g.,
 - Cannot spend more than \$100
 - Must be in Boston before 5:00PM



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Knapsack Problems



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Knapsack Problem

- You have limited strength, so there is a maximum weight knapsack that you can carry
- You would like to take more stuff than you can carry
- How do you choose which stuff to take and which to leave behind?
- Two variants
 - 0/1 knapsack problem
 - Continuous or fractional knapsack problem



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My Least-favorite Knapsack Problem



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0/1 Knapsack Problem, Formalized

- Each item is represented by a pair, $\langle \text{value}, \text{weight} \rangle$
- The knapsack can accommodate items with a total weight of no more than w
- A vector, L , of length n , represents the set of available items. Each element of the vector is an item
- A vector, V , of length n , is used to indicate whether or not items are taken. If $V[i] = 1$, item $I[i]$ is taken. If $V[i] = 0$, item $I[i]$ is not taken

0/1 Knapsack Problem, Formalized

Find a V that maximizes

$$\sum_{i=0}^{n-1} V[i] * I[i].value$$

subject to the constraint that

$$\sum_{i=0}^{n-1} V[i] * I[i].weight \leq w$$

Brute Force Algorithm

- 1. Enumerate all possible combinations of items. That is to say, generate all subsets of the set of items. This is called the **power set**.
- 2. Remove all of the combinations whose total units exceeds the allowed weight.
- 3. From the remaining combinations choose any one whose value is the largest.

Often Not Practical

- How big is power set?
- Recall
 - A vector, V , of length n , is used to indicate whether or not items are taken. If $V[i] = 1$, item $I[i]$ is taken. If $V[i] = 0$, item $I[i]$ is not taken
- How many possible different values can V have?
 - As many different binary numbers as can be represented in n bits
- For example, if there are 100 items to choose from, the power set is of size?
 - 1,267,650,600,228,229,401,496,703,205,376

Question 2

Are We Just Being Stupid?

- Alas, no
- 0/1 knapsack problem is inherently exponential
- But don't despair

Greedy Algorithm a Practical Alternative

- while knapsack not full
 - put “best” available item in knapsack
- But what does best mean?
 - Most valuable
 - Least expensive
 - Highest value/units

An Example

- You are about to sit down to a meal
- You know how much you value different foods, e.g., you like donuts more than apples
- But you have a calorie budget, e.g., you don't want to consume more than 750 calories
- Choosing what to eat is a knapsack problem

A Menu

Food	wine	beer	pizza	burger	fries	coke	apple	donut
Value	89	90	30	50	90	79	90	10
calories	123	154	258	354	365	150	95	195

- Let's look at a program that we can use to decide what to order

Class Food

```
class Food(object):
    def __init__(self, n, v, w):
        self.name = n
        self.value = v
        self.calories = w

    def getValue(self):
        return self.value

    def getCost(self):
        return self.calories

    def density(self):
        return self.getValue()/self.getCost()

    def __str__(self):
        return self.name + ': <' + str(self.value)\
               + ', ' + str(self.calories) + '>'
```

Build Menu of Foods

```
def buildMenu(names, values, calories):  
    """names, values, calories lists of same length.  
    name a list of strings  
    values and calories lists of numbers  
    returns list of Foods"""  
    menu = []  
    for i in range(len(values)):  
        menu.append(Food(names[i], values[i],  
                           calories[i]))  
    return menu
```

Implementation of Flexible Greedy

```
def greedy(items, maxCost, keyFunction):  
    """Assumes items a list, maxCost >= 0,  
        keyFunction maps elements of items to numbers"""  
    itemsCopy = sorted(items, key = keyFunction, ←  
                        reverse = True)  
  
    result = []  
    totalValue, totalCost = 0.0, 0.0  
  
    for i in range(len(itemsCopy)): ←  
        if (totalCost+itemsCopy[i].getCost()) <= maxCost: ←  
            result.append(itemsCopy[i])  
            totalCost += itemsCopy[i].getCost()  
            totalValue += itemsCopy[i].getValue()  
  
    return (result, totalValue)
```

Algorithmic Efficiency

```
def greedy(items, maxCost, keyFunction):  
→ itemsCopy = sorted(items, key = keyFunction,  
                      reverse = True)  
    result = []  
    totalValue, totalCost = 0.0, 0.0  
  
    for i in range(len(itemsCopy)): ←  
        if (totalCost+itemsCopy[i].getCost()) <= maxCost:  
            result.append(itemsCopy[i])  
            totalCost += itemsCopy[i].getCost()  
            totalValue += itemsCopy[i].getValue()  
  
    return (result, totalValue)
```

Question 3

Using greedy

```
def testGreedy(items, constraint, keyFunction):
    taken, val = greedy(items, constraint, keyFunction)
    print('Total value of items taken =', val)
    for item in taken:
        print('    ', item)
```

Using greedy

```
def testGreedy(maxUnits):  
    print('Use greedy by value to allocate', maxUnits,  
          'calories')  
    testGreedy(foods, maxUnits, Food.getValue)  
    print('\nUse greedy by cost to allocate', maxUnits,  
          'calories')  
    testGreedy(foods, maxUnits,  
                lambda x: 1/Food.getCost(x)) ←  
    print('\nUse greedy by density to allocate', maxUnits,  
          'calories')  
    testGreedy(foods, maxUnits, Food.density)  
  
testGreedy(800) ?
```

lambda

- lambda used to create anonymous functions
 - `lambda <id1, id2, ... idn>: <expression>`
 - Returns a function of n arguments
- Can be very handy, as here
- Possible to write amazing complicated lambda expressions
- **Don't**—use `def` instead

Using greedy

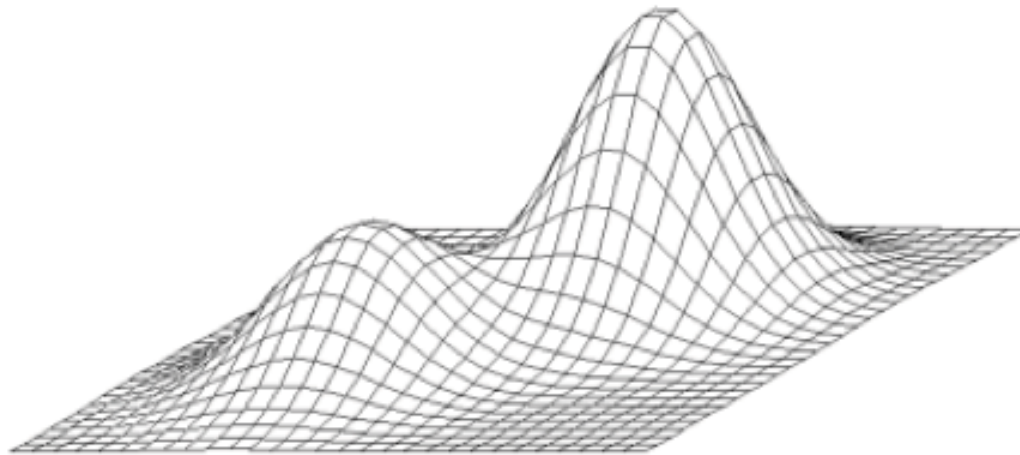
```
def testGreedy(foods, maxUnits):
    print('Use greedy by value to allocate', maxUnits,
          'calories')
    testGreedy(foods, maxUnits, Food.getValue)
    print('\nUse greedy by cost to allocate', maxUnits,
          'calories')
    testGreedy(foods, maxUnits,
                lambda x: 1/Food.getCost(x))
    print('\nUse greedy by density to allocate', maxUnits,
          'calories')
    testGreedy(foods, maxUnits, Food.density)

names = ['wine', 'beer', 'pizza', 'burger', 'fries',
         'cola', 'apple', 'donut', 'cake']
values = [89,90,95,100,90,79,50,10]
calories = [123,154,258,354,365,150,95,195]
foods = buildMenu(names, values, calories)
testGreedy(foods, 750)
```

[Run code](#)

Why Different Answers?

- Sequence of locally “optimal” choices don’t always yield a globally optimal solution



- Is greedy by density always a winner?
 - Try `testGreedy(foods, 1000)`

The Pros and Cons of Greedy

- Easy to implement
- Computationally efficient

- But does not always yield the best solution
 - Don't even know how good the approximation is
- In the next lecture we'll look at finding truly optimal solutions

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6.0002 Introduction to Computational Thinking and Data Science

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