

6.003: Signals and Systems

Signals and Systems

September 8, 2011

6.003: Signals and Systems

Today's handouts: Single package containing

- Slides for Lecture 1
- Subject Information & Calendar

Lecturer: Denny Freeman

Instructors: Elfar Adalsteinsson
Russ Tedrake

TAs: Phillip Nadeau
Wenbang Xu

Website: mit.edu/6.003

Text: *Signals and Systems* – Oppenheim and Willsky

6.003: Homework

Doing the homework is essential for understanding the content.

- where subject matter is/isn't learned
- equivalent to “practice” in sports or music

Weekly Homework Assignments

- Conventional Homework Problems plus
- **Engineering Design Problems** (Python/Matlab)

Open Office Hours !

- Stata Basement
- Mondays and Tuesdays, afternoons and early evenings

6.003: Signals and Systems

Collaboration Policy

- **Discussion** of concepts in homework is encouraged
- **Sharing** of homework or code is not permitted and will be reported to the COD

Firm Deadlines

- Homework must be submitted by the published due date
- Each student can submit **one** late homework assignment without penalty.
- Grades on other late assignments will be multiplied by 0.5 (unless excused by an Instructor, Dean, or Medical Official).

6.003 At-A-Glance

	Tuesday	Wednesday	Thursday	Friday
Sep 6	Registration Day: No Classes		R1: Continuous & Discrete Systems	R2: Difference Equations
Sep 13	L2: Discrete-Time Systems	HW1 due	R3: Feedback, Cycles, and Modes	R4: CT Systems
Sep 20	L4: CT Operator Representations	HW2 due	Student Holiday: No Recitation	R5: Laplace Transforms
Sep 27	L6: Z Transforms	HW3 due	R6: Z Transforms	R7: Transform Properties
Oct 4	L8: Convolution; Impulse Response	EX4	Exam 1 No Recitation	R8: Convolution and Freq. Resp.
Oct 11	Columbus Day: No Lecture	HW5 due	R9: Bode Diagrams	R10: Feedback and Control
Oct 18	L11: DT Feedback and Control	HW6 due	R11: CT Feedback and Control	R12: CT Feedback and Control
Oct 25	L13: CT Feedback and Control	HW7	Exam 2 No Recitation	R13: CT Fourier Series
Nov 1	L15: CT Fourier Series	EX8 due	R14: CT Fourier Series	R15: CT Fourier Transform
Nov 8	L17: CT Fourier Transform	HW9 due	R16: DT Fourier Transform	L18: DT Fourier Transform
Nov 15	L19: DT Fourier Transform	HW10	Exam 3 No Recitation	L20: Fourier Relations
Nov 22	L21: Sampling	EX11 due	R18: Fourier Transforms	Thanksgiving: No Lecture
Nov 29	L22: Sampling	HW12 due	R19: Modulation	L23: Modulation
Dec 6	L24: Modulation	EX13	R21: Review	L25: Applications of 6.003
Dec 13	Breakfast with Staff	EX13	R22: Review	Study Period: No Lecture
Dec 20	Final Examinations: No Classes			

6.003: Signals and Systems

Weekly meetings with **class representatives**

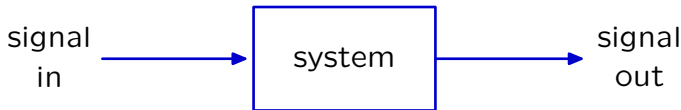
- help staff understand student perspective
- learn about teaching

Tentatively meet on Thursday afternoon

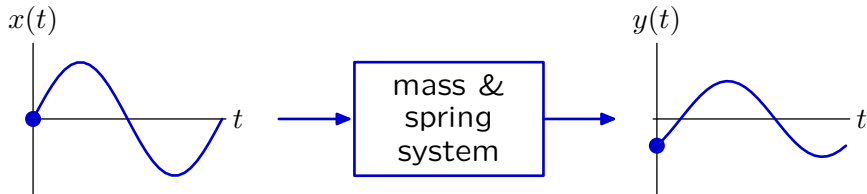
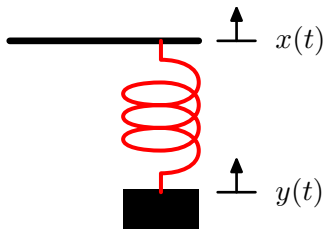
Interested? ...

The Signals and Systems Abstraction

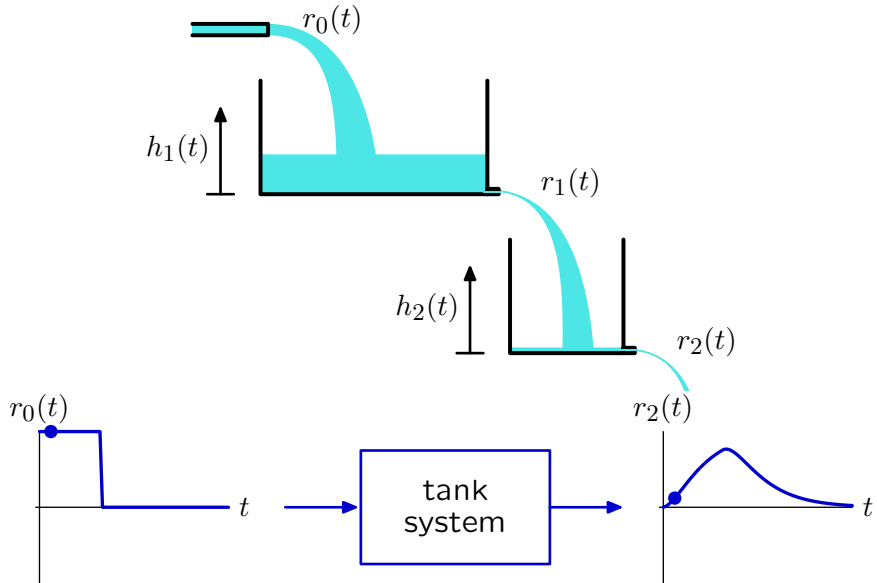
Describe a **system** (physical, mathematical, or computational) by the way it transforms an **input signal** into an **output signal**.



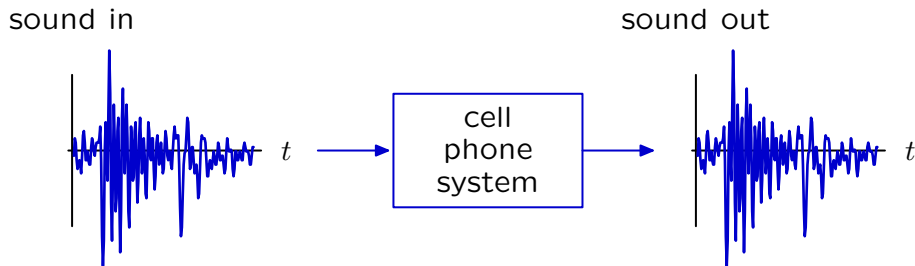
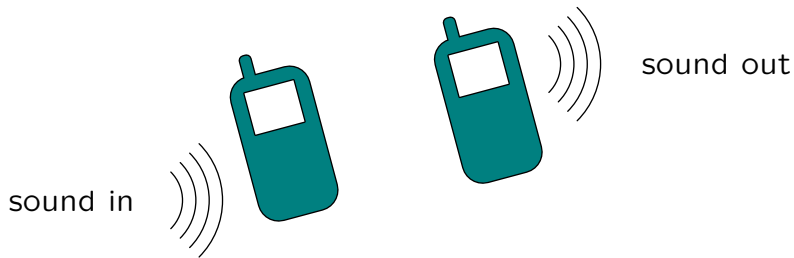
Example: Mass and Spring



Example: Tanks

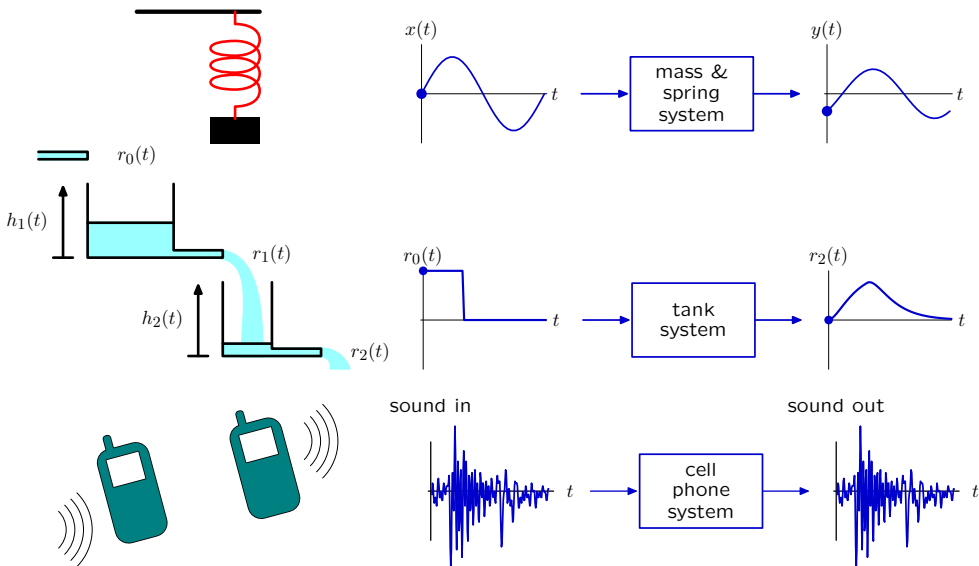


Example: Cell Phone System



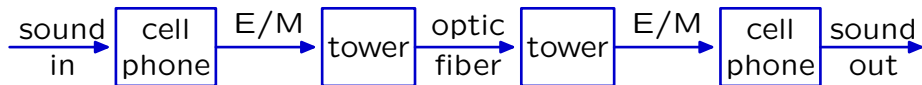
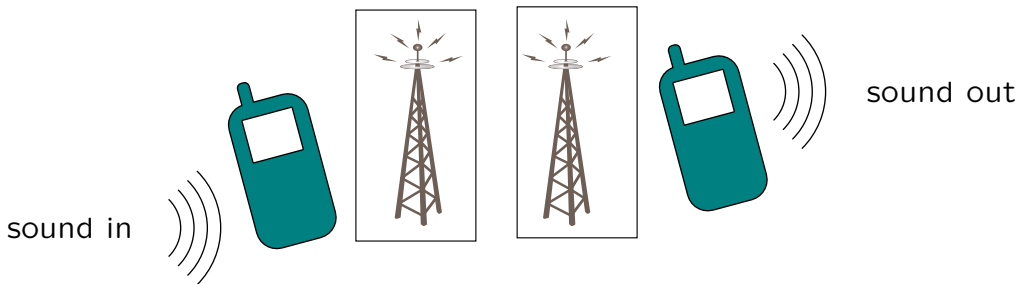
Signals and Systems: Widely Applicable

The Signals and Systems approach has broad application: electrical, mechanical, optical, acoustic, biological, financial, ...



Signals and Systems: Modular

The representation does not depend upon the physical substrate.

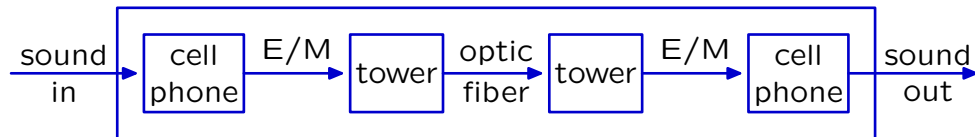


focuses on the flow of **information**, abstracts away everything else

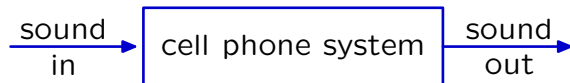
Signals and Systems: Hierarchical

Representations of component systems are easily combined.

Example: cascade of component systems



Composite system

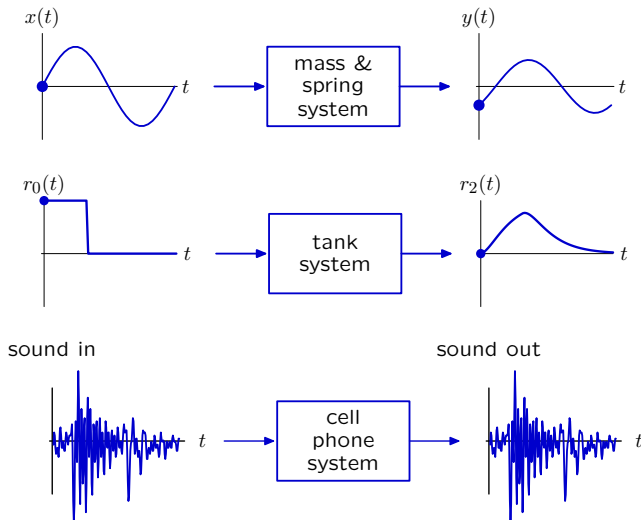


Component and composite systems have the same form, and are analyzed with same methods.

Signals and Systems

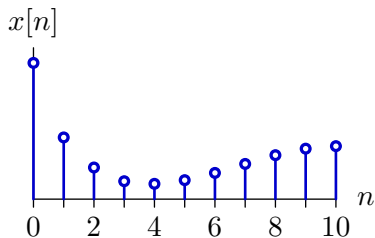
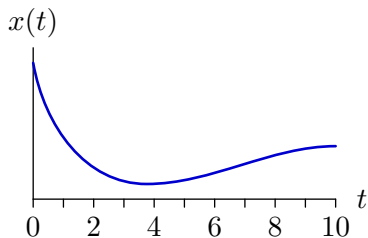
Signals are mathematical functions.

- independent variable = time
- dependent variable = voltage, flow rate, sound pressure



Signals and Systems

continuous “time” (CT) and discrete “time” (DT)



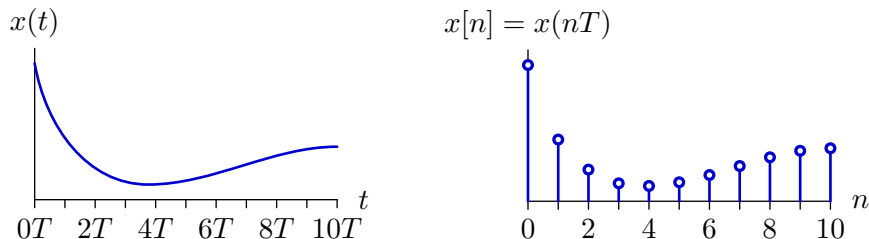
Signals from physical systems often functions of **continuous** time.

- mass and spring
- leaky tank

Signals from computation systems often functions of **discrete** time.

- state machines: given the current input and current state, what is the next output and next state.

Sampling: converting CT signals to DT



T = sampling interval

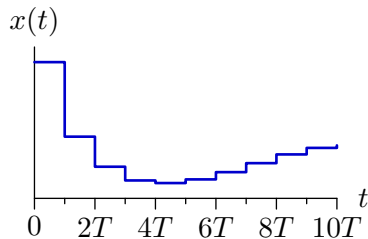
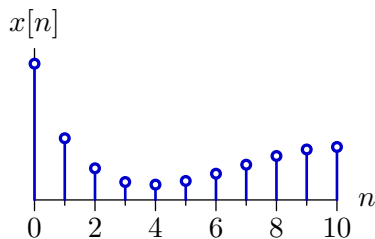
Important for computational manipulation of physical data.

- digital representations of audio signals (e.g., MP3)
- digital representations of images (e.g., JPEG)

Signals and Systems

Reconstruction: converting DT signals to CT

zero-order hold



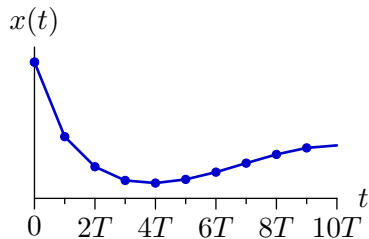
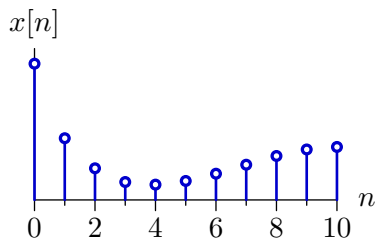
$T =$ sampling interval

commonly used in audio output devices such as CD players

Signals and Systems

Reconstruction: converting DT signals to CT

piecewise linear

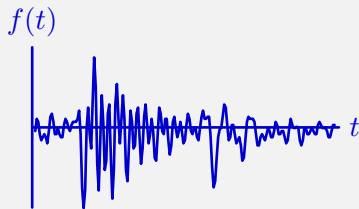


$T =$ sampling interval

commonly used in rendering images

Check Yourself

Computer generated speech (by Robert Donovan)



Listen to the following four manipulated signals:

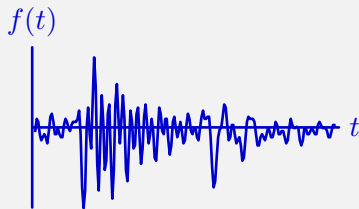
$$f_1(t), f_2(t), f_3(t), f_4(t).$$

How many of the following relations are true?

- $f_1(t) = f(2t)$
- $f_2(t) = -f(t)$
- $f_3(t) = f(2t)$
- $f_4(t) = \frac{1}{3}f(t)$

Check Yourself

Computer generated speech (by Robert Donovan)



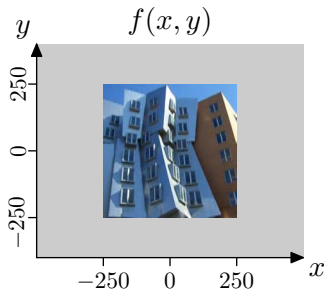
Listen to the following four manipulated signals:

$$f_1(t), f_2(t), f_3(t), f_4(t).$$

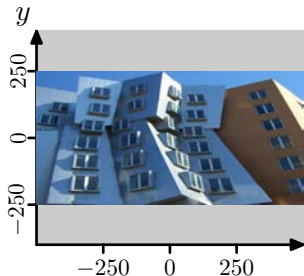
How many of the following relations are true? 2

- $f_1(t) = f(2t)$ ✓
- $f_2(t) = -f(t)$ ✗
- $f_3(t) = f(2t)$ ✗
- $f_4(t) = \frac{1}{3}f(t)$ ✓

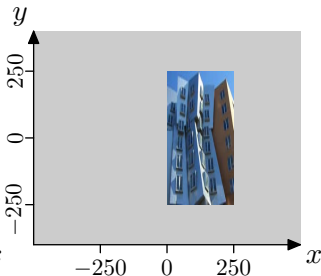
Check Yourself



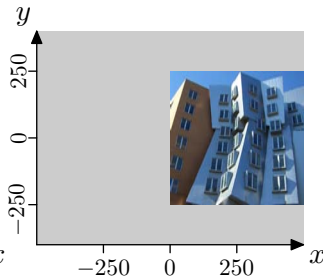
How many images match the expressions beneath them?



$$f_1(x, y) = f(2x, y) ?$$

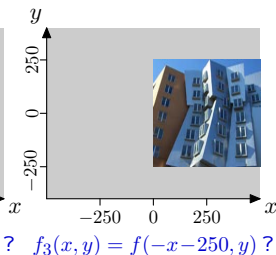
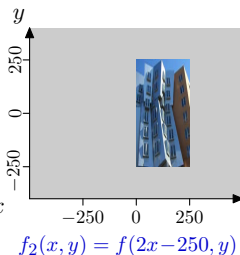
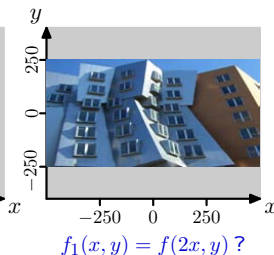
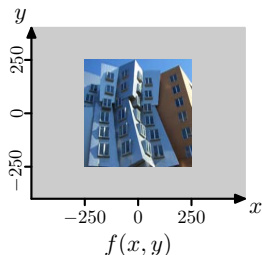


$$f_2(x, y) = f(2x - 250, y) ?$$



$$f_3(x, y) = f(-x - 250, y) ?$$

Check Yourself



$$x = 0 \rightarrow f_1(0, y) = f(0, y) \quad \checkmark$$

$$x = 250 \rightarrow f_1(250, y) = f(500, y) \quad \times$$

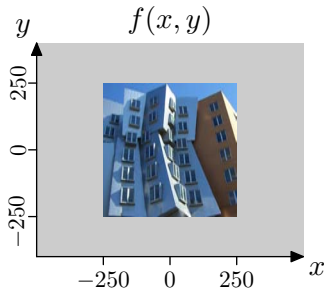
$$x = 0 \rightarrow f_2(0, y) = f(-250, y) \quad \checkmark$$

$$x = 250 \rightarrow f_2(250, y) = f(250, y) \quad \checkmark$$

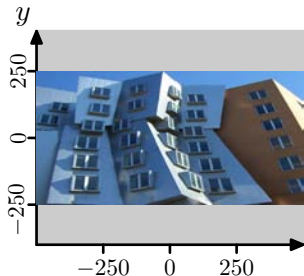
$$x = 0 \rightarrow f_3(0, y) = f(-250, y) \quad \times$$

$$x = 250 \rightarrow f_3(250, y) = f(-500, y) \quad \times$$

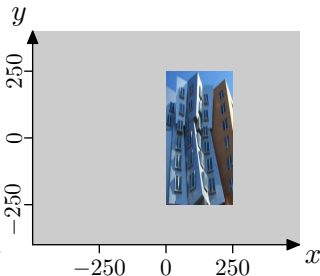
Check Yourself



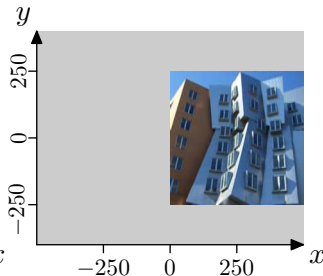
How many images match the expressions beneath them?



~~$f_1(x, y) = f(2x, y) ?$~~



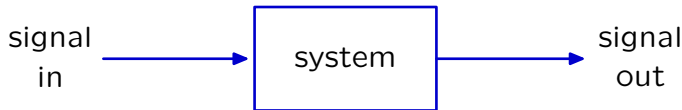
$f_2(x, y) = f(2x - 250, y) ?$



~~$f_3(x, y) = f(x - 250, y) ?$~~

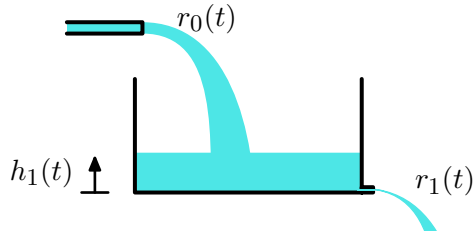
The Signals and Systems Abstraction

Describe a **system** (physical, mathematical, or computational) by the way it transforms an **input signal** into an **output signal**.



Example System: Leaky Tank

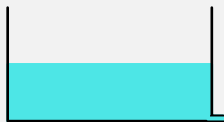
Formulate a mathematical description of this system.



What determines the leak rate?

Check Yourself

The holes in each of the following tanks have equal size. Which tank has the largest leak rate $r_1(t)$?



1.



2.



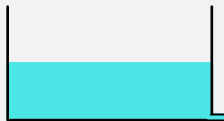
3.



4.

Check Yourself

The holes in each of the following tanks have equal size.
Which tank has the largest leak rate $r_1(t)$? 2



1.



2.



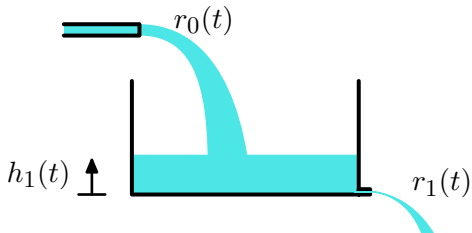
3.



4.

Example System: Leaky Tank

Formulate a mathematical description of this system.

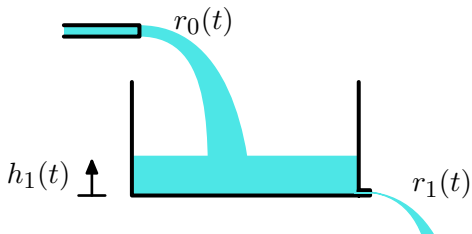


Assume linear leaking: $r_1(t) \propto h_1(t)$

What determines the height $h_1(t)$?

Example System: Leaky Tank

Formulate a mathematical description of this system.



Assume linear leaking: $r_1(t) \propto h_1(t)$

Assume water is conserved: $\frac{dh_1(t)}{dt} \propto r_0(t) - r_1(t)$

Solve: $\frac{dr_1(t)}{dt} \propto r_0(t) - r_1(t)$

Check Yourself

What are the dimensions of constant of proportionality C ?

$$\frac{dr_1(t)}{dt} = C(r_0(t) - r_1(t))$$

Check Yourself

What are the dimensions of constant of proportionality C ?
inverse time (to match dimensions of dt)

$$\frac{dr_1(t)}{dt} = C(r_0(t) - r_1(t))$$

Analysis of the Leaky Tank

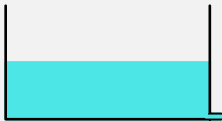
Call the constant of proportionality $1/\tau$.

Then τ is called the **time constant** of the system.

$$\frac{dr_1(t)}{dt} = \frac{r_0(t)}{\tau} - \frac{r_1(t)}{\tau}$$

Check Yourself

Which tank has the largest time constant τ ?



1.



2.



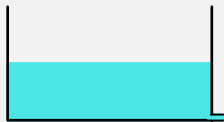
3.



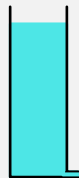
4.

Check Yourself

Which tank has the largest time constant τ ? 4



1.



2.



3.



4.

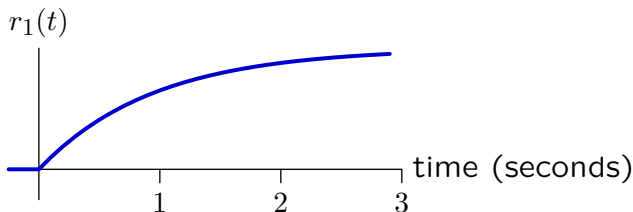
Analysis of the Leaky Tank

Call the constant of proportionality $1/\tau$.

Then τ is called the **time constant** of the system.

$$\frac{dr_1(t)}{dt} = \frac{r_0(t)}{\tau} - \frac{r_1(t)}{\tau}$$

Assume that the tank is initially empty, and then water enters at a constant rate $r_0(t) = 1$. Determine the output rate $r_1(t)$.



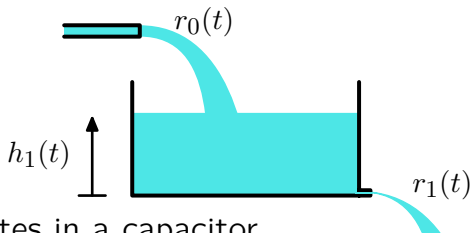
Explain the shape of this curve mathematically.

Explain the shape of this curve physically.

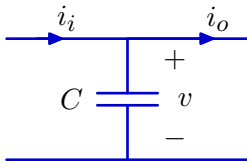
Leaky Tanks and Capacitors

Although derived for a leaky tank, this sort of model can be used to represent a variety of physical systems.

Water accumulates in a leaky tank.



Charge accumulates in a capacitor.



$$\frac{dv}{dt} = \frac{i_i - i_o}{C} \propto i_i - i_o$$

analogous to

$$\frac{dh}{dt} \propto r_0 - r_1$$

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